Aerospace Radar

- Lesson 3: AEROSPACE RADAR BASIC RELATIONS

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AEROSPACE RADAR BASIC RELATIONS

Definition of basic angles

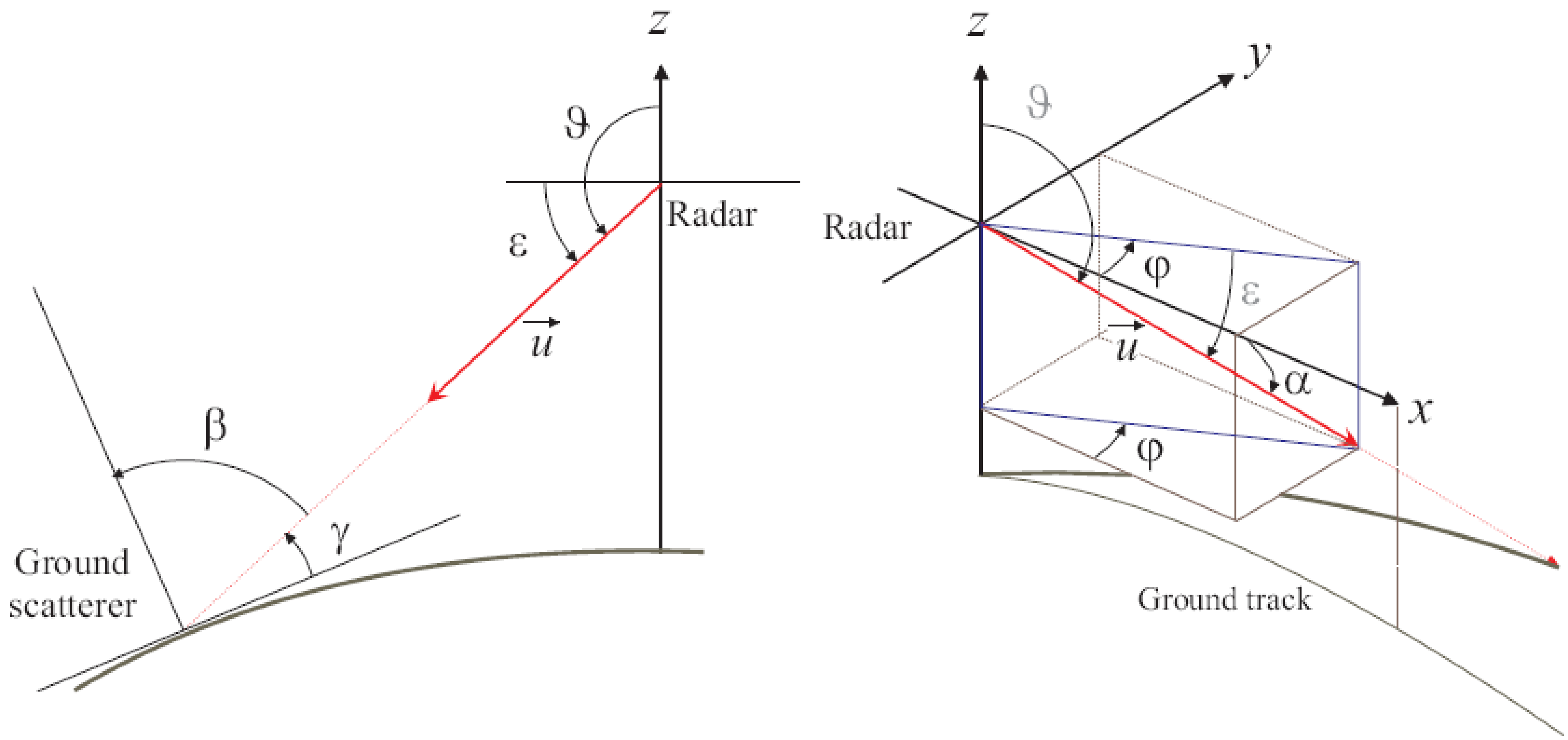


Figure 4.1: *Left: Definition of angles in the vertical plane:* β*: Incidence angle,* ϵ*: Depression angle,* γ*: Grazing angle,* ϑ*: Angle used for spherical coordinate system. Right:* φ*: Azimuth angle,* α*: Cone angle*

AEROSPACE RADAR BASIC RELATIONS
Definition of basic angles

ELEVATION

- β Incidence angle
- ε Depression angle
- $\gamma = \pi/2-\varepsilon$ Grazing angle

AZIMUTH

- φ Azimuth angle

Relative to motion axis

- α Cone angle

- **For flat earth:**

$$\gamma = \varepsilon$$

- **Directional cosine rel. to x-axis**

$$\cos\alpha = \cos\varphi\cos\varepsilon$$

AEROSPACE RADAR BASIC RELATIONS

Doppler frequency

The airplane flies with velocity V in direction of the x-axis, velocity vector $\vec{V}$

- Line-of-sight vector to an object (in platform coordinates)

$$\vec{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \|\vec{u}\| = 1$$

- Radial velocity of an earth fixed scattering center

$$v_r = -\langle \vec{V}, \vec{u} \rangle = -V \cos\alpha = -Vu$$

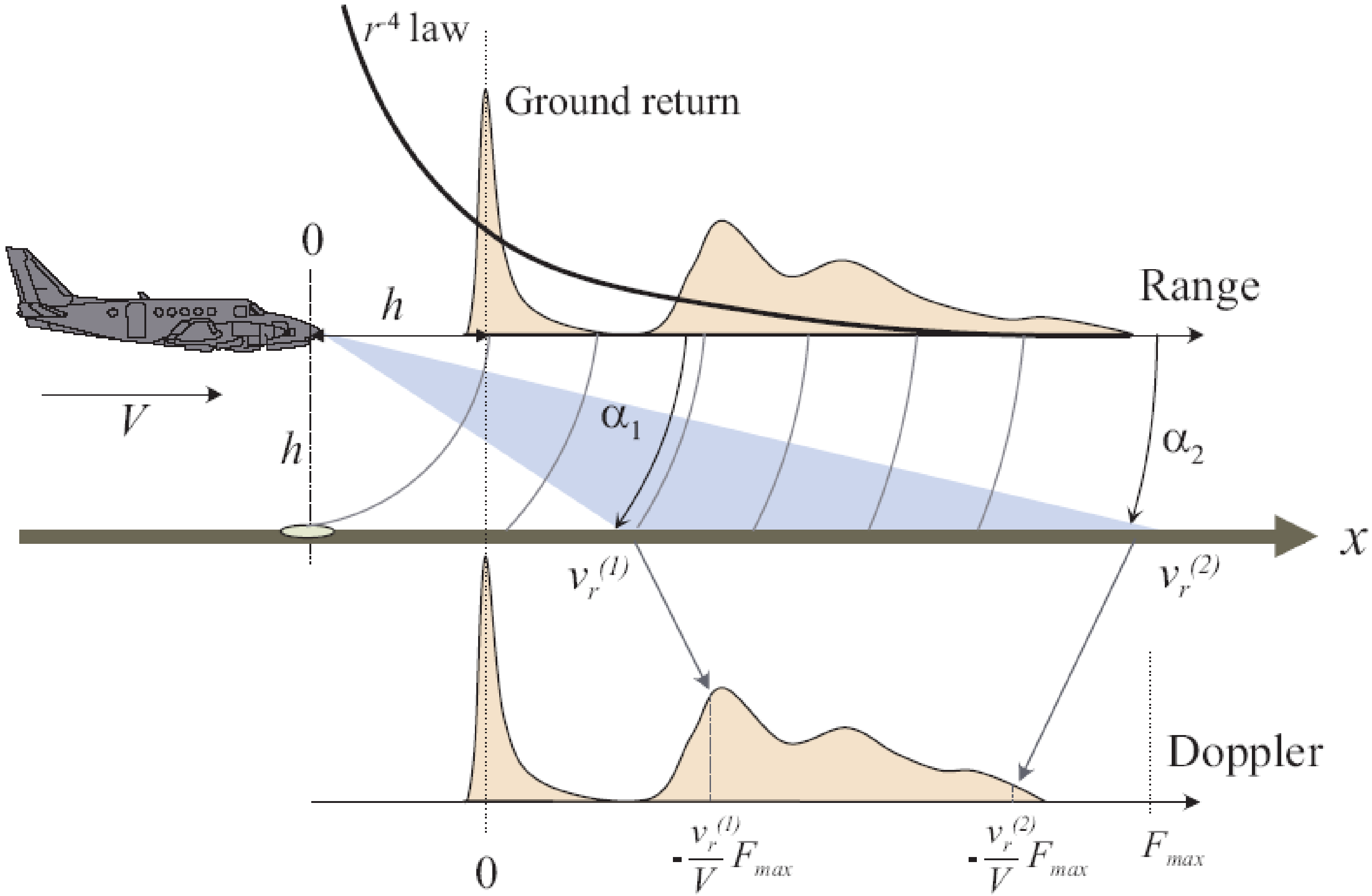
- Doppler frequency

$$F = -\frac{2v_r}{\lambda} = \frac{2V}{\lambda} u = uF_{\max}$$

$$F_{\max} = \frac{2V}{\lambda}$$

is the maximum magnitude of Doppler frequencies induced by earth-fixed objects

AEROSPACE RADAR BASIC RELATIONS
Energy distribution in range and Doppler



AEROSPACE RADAR BASIC RELATIONS

Energy distribution in range and Doppler

Range:

- If the flight altitude is *h* the first echo appears at range *r*=*h*: *Nadir return*
- *Specular reflection*, very strong
- The mean echo power is decreasing proportional to r^{-4}, modulated by the two-way antenna elevation characteristics

Doppler:

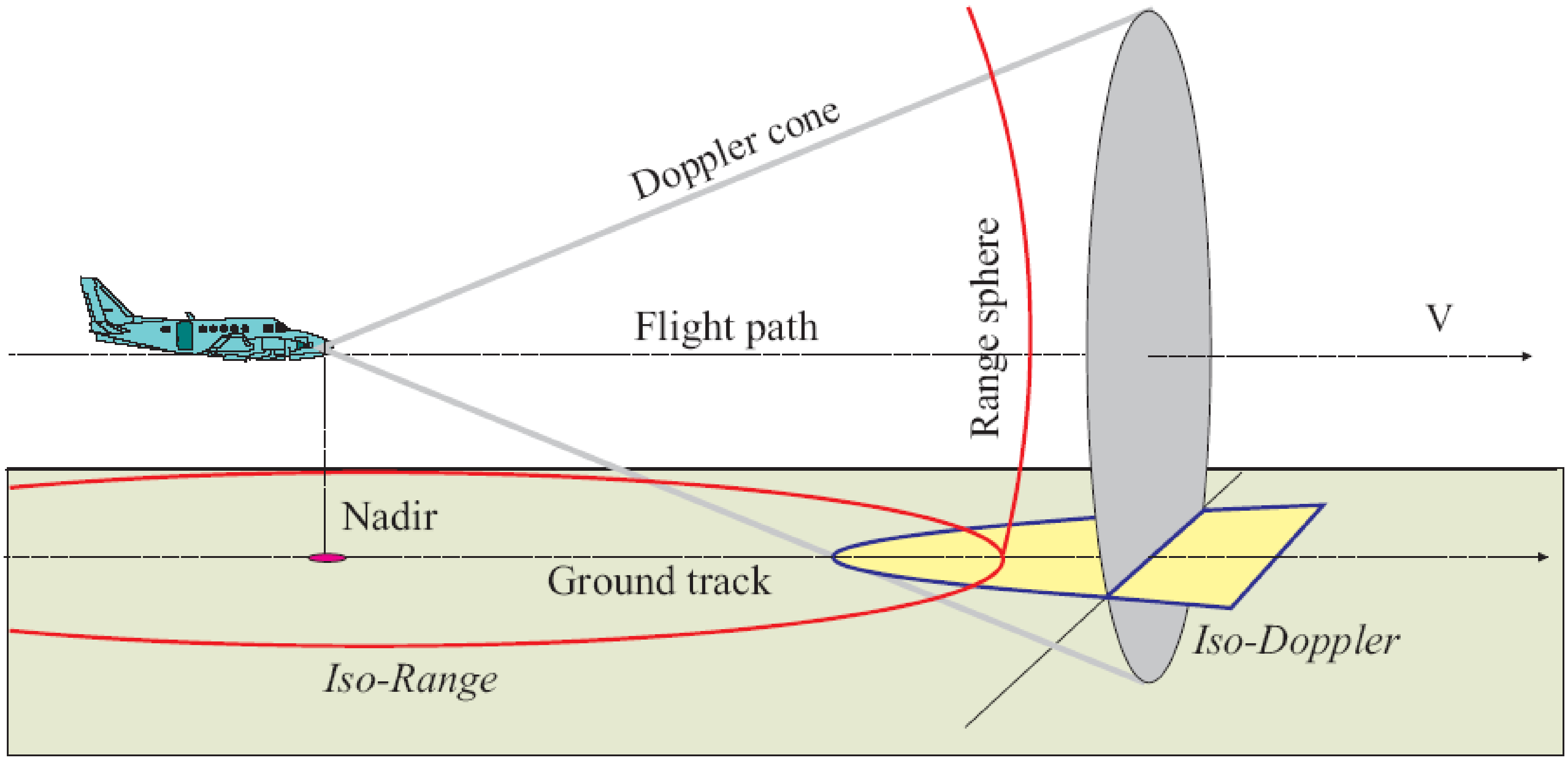
- The Doppler frequency of a ground fixed scatterer at range *r* is given by

$$\begin{aligned} F &= uF_{\max} = \cos\alpha\, F_{\max} \\ &= \cos\varepsilon \cos\varphi\, F_{\max} \\ &= \frac{\sqrt{r^2 - h^2}}{r} \cos\varphi\, F_{\max} \end{aligned}$$

- Energy distribution according to r^{-4} with the range corresponding to Doppler
- Nadir return at Doppler = 0
- Doppler spread of *clutter* within the main beam
  $\Rightarrow$ Difficulty to detect moving targets

AEROSPACE RADAR BASIC RELATIONS

ISO-Range and ISO-Doppler contours



Range sphere and Doppler cone

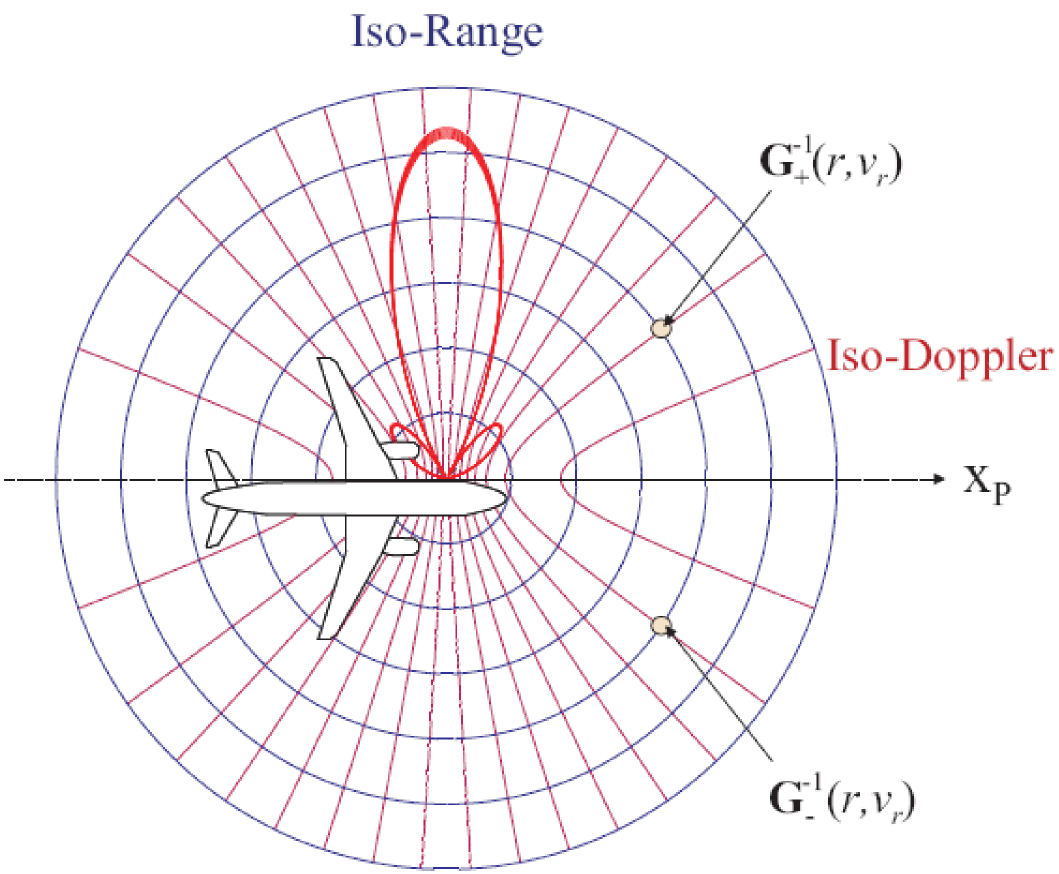
ISO-Range and ISO-Doppler contours

The ISO-range lines on the surface are circles centered at the nadir, the ISO-Doppler lines are cuts of the Doppler-cone with the surface.

The Doppler-cone has its axis in flight direction and its corner at the antennas phase center, the cone angle α is related to the radial velocity by $v_r = -V \cos \alpha$.

AEROSPACE RADAR BASIC RELATIONS

Mapping between range - radial velocity and earth surface



- Any point $(x,y)^t$ at the earth surface produces an echo at range-velocity $(r,v_r)^t = \mathbf{G}(x,y)$
- Vice versa: For each range-velocity pair $(r,v_r)^t$ there are two points $(x,y)^t$ at the earth surface producing an echo at range-velocity $(r,v_r)^t$

AEROSPACE RADAR BASIC RELATIONS

Mapping between range - radial velocity and earth surface

- Range-velocity pair induced by a ground scatterer at $(x,y)^t$:

$$\mathbf{G}(x,y) \;=\; \begin{pmatrix} r \\ v_r \end{pmatrix}(x,y) = \begin{pmatrix} \sqrt{x^2+y^2+h^2} \\ -V\frac{x}{\sqrt{x^2+y^2+h^2}} \end{pmatrix}.$$

- Position (x,y) of ground scatterer with range velocity pair $(r,v_r)^t$:

$$\mathbf{G}_{\pm}^{-1}(r,v_r) \;=\; \begin{pmatrix} x \\ y_{\pm} \end{pmatrix}(r,v_r) = \begin{pmatrix} -r\frac{v_r}{V} \\ \pm\sqrt{r^2\left(1-\left(\frac{v_r}{V}\right)^2\right)-h^2} \end{pmatrix}$$

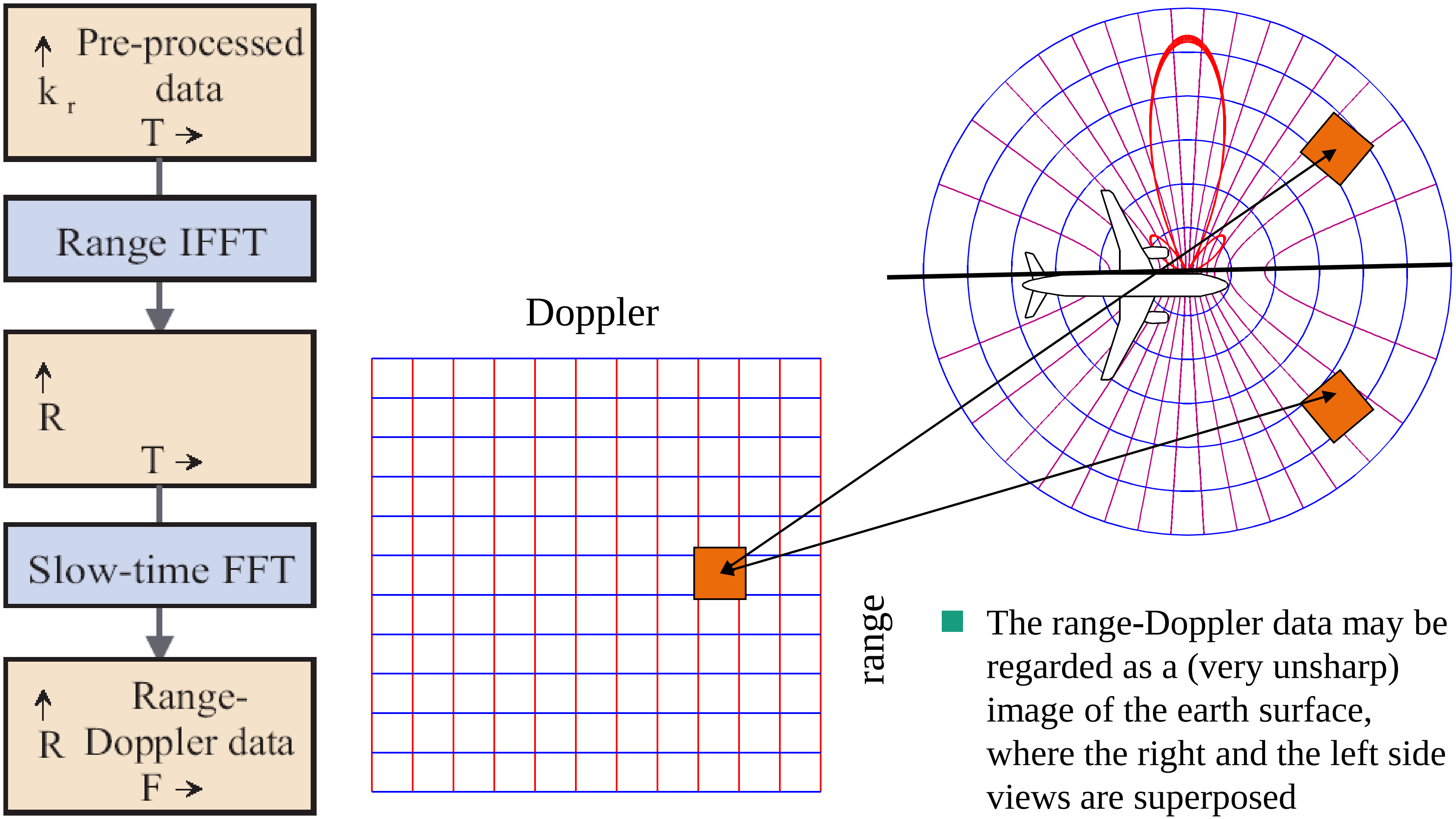
- If the Doppler is unambiguos, there is an unique relation between radial velocity and Doppler by

$$v_r = -\frac{F}{F_{\max}}V, \quad F = -\frac{v_r}{V}F_{\max}$$

For each range-velocity pair (r, v_r) there are two points on the earth surface generating echo signals at (r, v_r). Their positions are symmetric to the flight path projected on the ground.

AEROSPACE RADAR BASIC RELATIONS

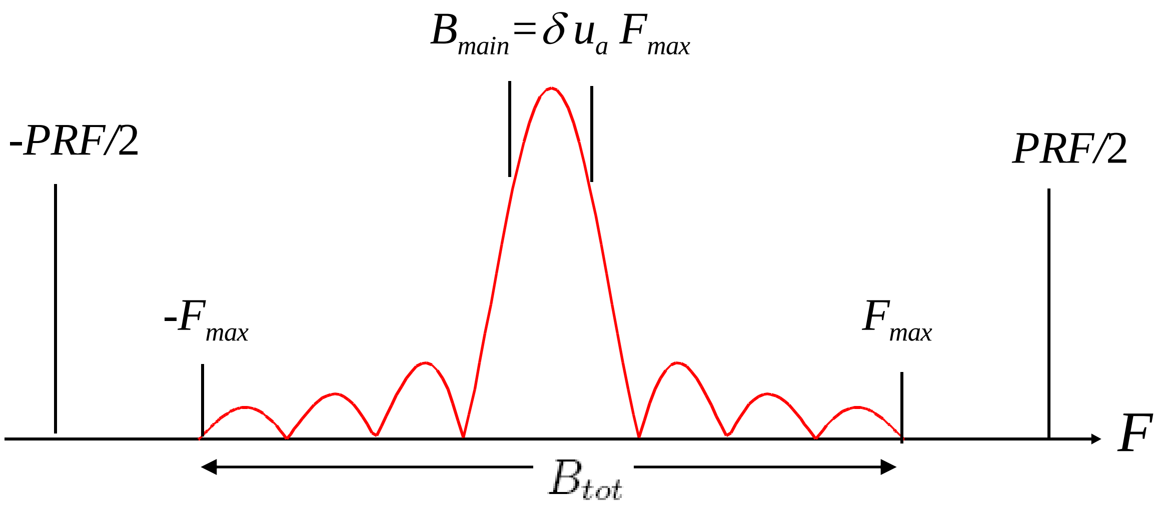
Mapping between range - radial velocity and earth surface



- The range-Doppler data may be regarded as a (very unsharp) image of the earth surface, where the right and the left side views are superposed

AIRBORNE RADAR BASIC

Doppler spectrum of clutter



$$F_{\max} = \frac{2V}{\lambda}$$

$$P_C(F) \approx const \left| D\left(\frac{F}{F_{max}}\right) \right|^2$$

- *The mainbeam clutter.* It's bandwidth is given by $B_{main} = \delta u_a F_{max}$, where δu_a denotes the beamwidth of the two-way characteristics.

- *The sidelobe clutter.* The spectrum is sharply bounded by the condition $-1 \leq u \leq 1$. So we have for the bandwidth of the whole clutter spectrum: $B_{tot} = 2F_{max}$.

- *The clutter free region.* This is outside the bandwidth B_{tot}.

AEROSPACE RADAR BASIC RELATIONS

Clutter spectrum

- For most airborne radar systems, it is impossible to cover all of the sidelobe clutter or even parts of the clutter free region unambiguously by the sampling frequency $F_s = PRF$.
- This can be achieved only if $\Delta T > 1/B_{tot} = \lambda/(4V)$, i.e. the pulse repetition interval has to be shorter than the time needed by the platform to fly the distance of $\lambda/4$!
- For GHz frequencies and normal airspeed this would require a PRF larger than allowed to avoid range ambiguities.
- As a consequence, normally at least the sidelobe part of the clutter will be aliased, so there will hardly be any completely clutter-free region!
- Common SAR systems apply an azimuth sampling frequency equal to or a little above B_{main}.
- MTI-systems should use a considerably higher PRF!

AEROSPACE RADAR BASIC RELATIONS

Condition sampling unambiguously in the main beam clutter band

- To sample the main beam clutter according to the Nyquist criterium $F_s = PRF$ has to be larger or equal to B_{main}.

$$PRF = \frac{1}{\Delta T} \geq B_{main}$$

$$\Delta T \leq \frac{1}{B_{main}} = \frac{1}{\delta u_a F_{\max}} = \frac{\lambda}{2\delta u_a V}$$

$$V\Delta T \leq \frac{\lambda}{2\delta u_a}$$

- Condition for the way allowed to fly between two pulses!

- An antenna of length l_x has the beamwidth

$$\delta u_a = \frac{\lambda}{l_x}$$

It follows

$$V\Delta T \leq \frac{l_x}{2}$$

- so the platform should not move more than a half antenna length between two pulses.

AEROSPACE RADAR BASIC RELATIONS

Range ambiguities

$$\Delta r = \frac{c_0}{2}\Delta T$$

- Slant range ambiguities

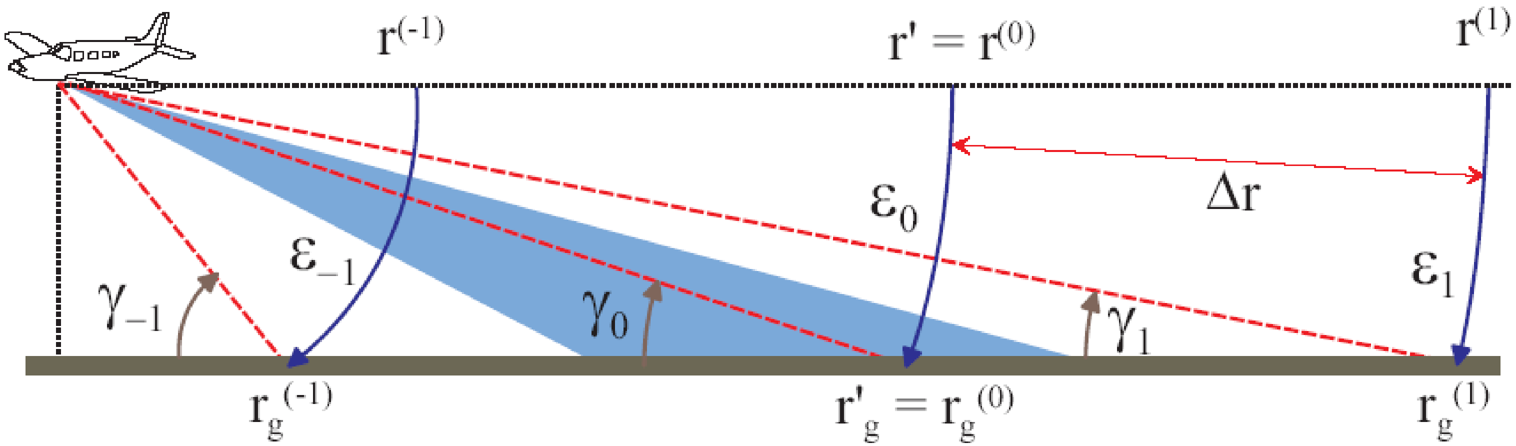
$$r_\nu = r + \nu \Delta r$$

- Ground range ambiguities

$$r_g^{(\nu)} = \sqrt{r_\nu^2 - h^2}$$

- Depression angle ambiguities

$$\epsilon_\nu = \arcsin\left(\frac{h}{r_\nu}\right)$$



Cone ambiguities

- Doppler ambiguities

$$\Delta F = 1/\Delta T$$

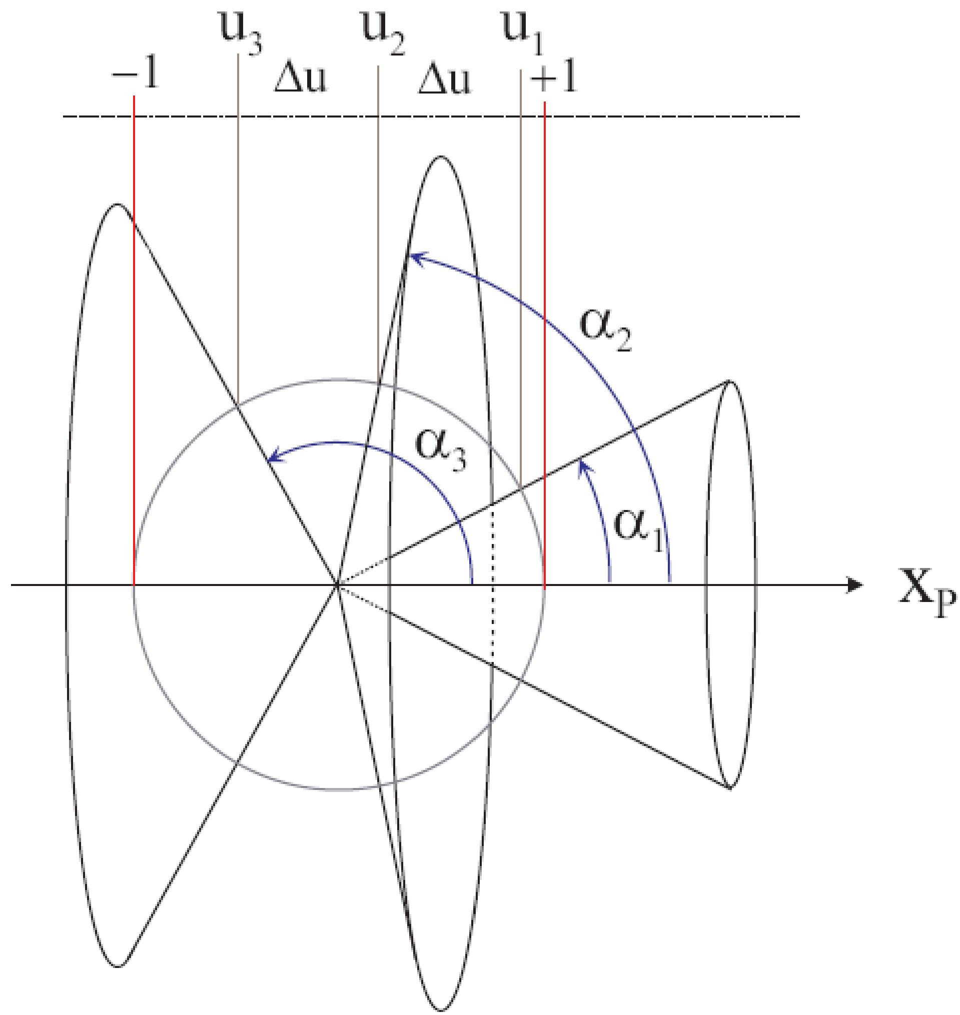
- Radial velocity ambiguities

$$\Delta v = \lambda/(2\Delta T)$$

$$v_\mu = v_r + \mu\Delta v$$

- Directional cosine ambiguities

$$\Delta u = \frac{\Delta v}{V} = \frac{\lambda}{2V\Delta T} = \frac{\lambda}{2\Delta x}$$



Ambiguous Doppler cones

AEROSPACE RADAR BASIC RELATIONS

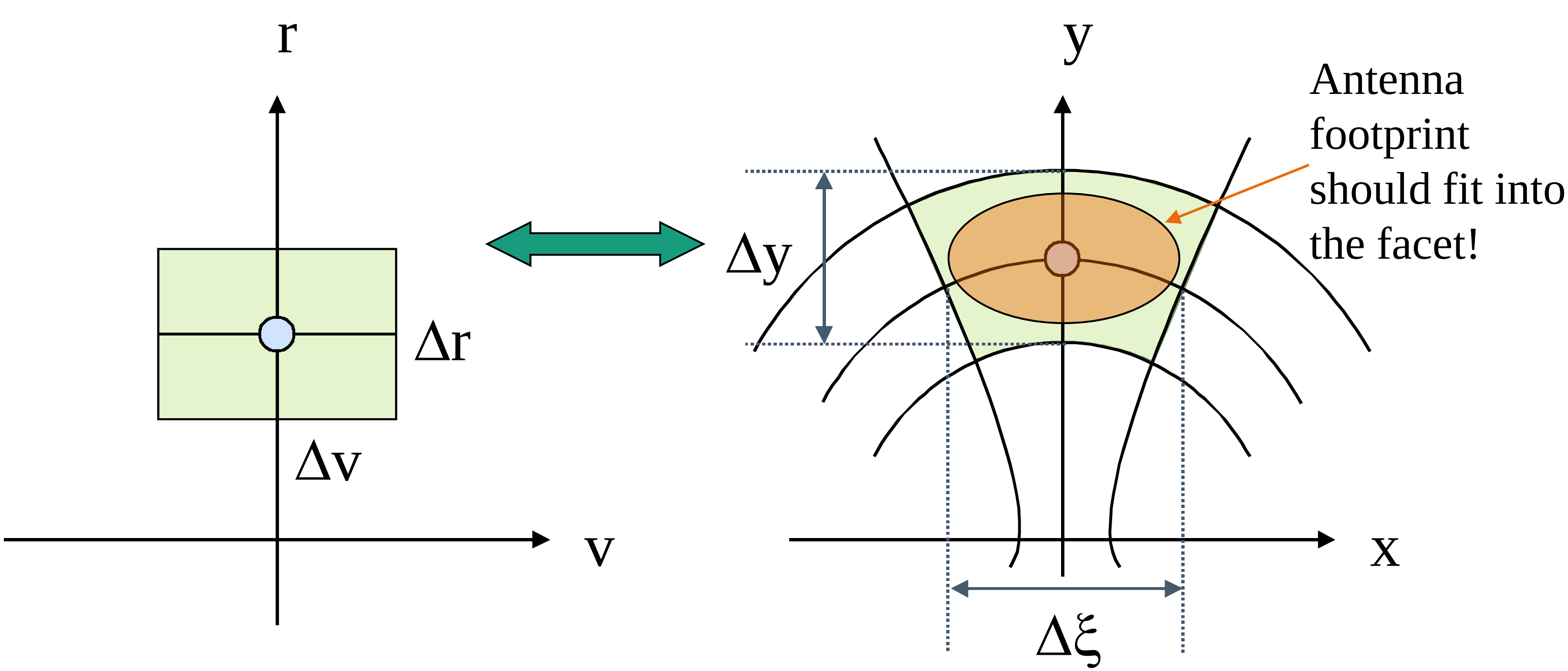
Ambiguity facet in the sidelooking configuration

$$A_{vr} = \Delta v \Delta r = \frac{\lambda c_0}{4}$$

(independent on PRF)

$$\Delta\xi \approx r\frac{\Delta v}{V} \text{ and } \Delta y \approx \frac{\Delta r}{\cos\epsilon}$$

$$A_{xy} = \Delta\zeta\Delta y = \frac{r}{V\cos\varepsilon}A_{vr} = \frac{r\lambda c_0}{4V\cos\varepsilon}$$



AEROSPACE RADAR BASIC RELATIONS
Ambiguity facet

- Area of the ambiguity facet on ground

$$A_{xy} = \frac{r\lambda c_0}{4V\cos\varepsilon}$$

- Numerical example for space based radar

r	700 km
V	7600 m/s
λ	3 cm
ε	45 deg

→

A_{xy}	293 km^2

Design considerations for a space based SAR:

- Flight altitude should be about 500 km to avoid interactions with the atmosphere
- It follows a velocity of about 7600 m/s
- The depression angle should not be to low (shadowing, large range) and not too large (bad ground resolution)
- The frequency should be between L-band and Ku-band

$\Rightarrow$ Ambiguity area cannot be changed too much!

AEROSPACE RADAR BASIC RELATIONS

Minimum antenna area

- Beamwidths of an antenna with length l_x and height l_z (area $a = l_x l_z$)

$$\delta u = \frac{\lambda}{l_x} \quad \delta w = \frac{\lambda}{l_z}$$

- Spatial angle of main beam

$$\Omega = \delta u \, \delta w = \frac{\lambda^2}{l_x l_z} = \frac{\lambda^2}{a}$$

- Illuminated area at range r perpendicular to look direction

$$A_\perp = \Omega r^2 = r^2 \frac{\lambda^2}{a}$$

- Area of antenna footprint

$$A_{foot} = \frac{A_\perp}{\sin \varepsilon} = \frac{\lambda^2 r^2}{a \sin \varepsilon}$$

- Ambiguity area

$$A_{xy} = \frac{r \lambda c_0}{4V \cos \varepsilon}$$

- Condition to avoid ambiguities

$$A_{foot} \leq A_{xy}$$

$$\frac{\lambda^2 r^2}{a \sin \varepsilon} \leq \frac{r \lambda c_0}{4V \cos \varepsilon}$$

$$a \geq \frac{\lambda^2 r^2}{\sin \varepsilon} \frac{4V \cos \varepsilon}{r \lambda c_0} = \frac{4 \lambda r V}{c_0 \tan \varepsilon}$$

Mapping between range - radial velocity and earth surface

If range and azimuth ambiguities shall be avoided by the illumination by the antenna main beam, the antenna area has to fulfill the inequation:

$$a \geq \frac{4\lambda r V}{c_0 \tan \varepsilon}$$

- Numerical example for space based radar

r	700 km
V	7600 m/s
λ	3 cm
ε	45 deg

→

a_{min}	2.1m^2

- In order to fit the whole main beam from zero to zero into the ambiguity area, the dimensions of the antenna have to be increased by a factor two, i. e. the antenna area has to be four times as large; for the example we get a minimum area of 8.5 m^2.