Aerospace Radar

- Lesson 4: SYNTHETIC APERTURE RADAR

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SYNKETIC APERTURE RADAR

Imaging radar (SAR, ISAR) is based on...

- Measurement of range change via phase
  (-> cross range resolution)

- Relative aspect change of the scene, the object necessary

- SAR: via motion of the platform

- ISAR: via motion of the target
Spatial Fourier transformation

Let \( a(x) \) be a complex function of the spatial variable \( x \)

One dimensional spatial Fourier transformation

\[
A(K_x) := \int a(x)e^{-jK_xx} \, dx
\]

\( K_x \) is a spatial frequency [rad/m] which represents the phase variation along the space.

- \( d \)-dimensional spatial Fourier transformation

\[
A(K) := \int a(r)e^{-jK^t r} \, dr
\]

- Inverse spatial Fourier transformation

\[
a(r) = \frac{1}{(2\pi)^d} \int A(K)e^{jK^t r} \, dK
\]

Examples

1) \( a(r) = \delta(r - r_0) \)
   \[
   A(K) = e^{-jK^t r_0}
   \]
   Most rapid phase change in direction of \( r_0 \)

2) \( A(K) = \delta(K - K_0) \)
   \[
   a(r) = \frac{1}{(2\pi)^d} e^{jK_0^t r}
   \]
   Plane wave with \( \lambda = 2\pi/|K_0| \)
2D Fourier correspondence

\[ a(r) = \delta(r - r_0) \]

\[ A(K) = e^{-jK^t r_0} \]
Restriction to a $K$-set

Back transformation over a $K$-set

$$\hat{a}_K(r) = \frac{1}{(2\pi)^d} \int_K A(K) e^{jK^t r} dK$$

yields an unsharp reconstruction of $a(r)$

- Different choices of the $K$-set effect different (in phase and speckle) unsharp reconstructions.

- In the SAR-context the generation of multiple images via different $K$-sets is called 'Multilook'.

- Incoherent summation of the unsharp reconstructions reduces the speckle.
Restriction to a $K$-set

Fourier transformation

Back Transf.

Restriction to $K$-sets
Radar imaging with turn table

Range histories of isolated scatterers
Geometry for the turn table experiment

\[
\begin{align*}
R(\varphi; x, y) &= \sqrt{(R_0 \cos \varphi + x)^2 + (R_0 \sin \varphi + y)^2} \\
&\approx R_0 + (x \cos \varphi + y \sin \varphi) \\
&= R_0 + u^t(\varphi)r.
\end{align*}
\]

Far field approximation!

\[
\mathbf{u}(\varphi) = \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix}
\]

\[
\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}
\]
Radar signals from the turntable experiment

Signal from a single scatterer at \((x,y)\) after range pre-processing

\[
S(k_r, \varphi; x, y) = e^{-jk_r(2(R(\varphi;x,y)-R_{ref})}
\]

- Superposed signals, reflectivity \(a(x,y)\)

\[
Z(k_r, \varphi) = \int \int S(k_r, \varphi; x, y)a(x, y)\,dx\,dy
\]

\[
Z(k_r, \varphi) \approx \int \int e^{-j2k_r(x\cos\varphi+y\sin\varphi)}a(x, y)\,dx\,dy
\]

- Define:

\[
K_x := 2k_r \cos \varphi; \quad K_y := 2k_r \sin \varphi
\]

- Variable substitution (Re-formatting)

\[
Z(K_x, K_y) = \int e^{-j(K_xx+K_yy)}a(x, y)\,dx\,dy = A(K_x, K_y)
\]
Polar Reformatting

To apply a DFT, the measurements on a polar grid have to be interpolated to a rectangular grid.
Point spread function

Measured $K$-set:

$$\mathcal{K} = \{2k_r (\cos \varphi, \sin \varphi)^t : k_r \in [k_1, k_2], \varphi \in [\varphi_1, \varphi_2]\}$$

$$\hat{a}(x, y) = \frac{1}{(2\pi)^2} \int \int_{\mathcal{K}} z(K_x, K_y) e^{j(K_x x + K_y y)} dK_x dK_y$$

$$a(r) = \delta(r - r_0)$$

$$\hat{a}(r) = \frac{1}{(2\pi)^d} \int_{\mathcal{K}} exp\{jK^t(r - r_0)\} dK$$

- The point spread function is given by the Fourier transform of the Indicator function of the $K$-set
Resolution

Extension of the K-set in x- and y-direction (for small angles):

\[
\Delta K_x = 2 \sin(\varphi/2) \bar{K} \\
\Delta K_y = \Delta K
\]

- Resolution:

\[
\delta x = \frac{2\pi}{\Delta K_x} = \frac{\bar{\lambda}}{4 \sin(\Delta \varphi/2)} \\
\delta y = \frac{2\pi}{\Delta K_y} = \frac{c}{2b}
\]
2D FFT

FFT columns, \( x S^*(f) \)

IFFT columns

Polar Reformattng

2D IFFT

(Amplitude)
Turntable ISAR-imaging at 94 Ghz
Turntable ISAR-imaging at 220 Ghz (8 GHz bandwidth)
ISAR film of rotating TIRA antenna
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Applications

- **Geology, Cartography:** Chartering of areas which are permanently under dense cloud cover, surveying of geological structures, worldwide surveying of human settlements and cultivated land.

- **Oceanography:** Analysis of ocean waves, conclusions on wind fields, research on tsunamis.

- **Hydrology:** Surveying of floods and soil moisture.

- **Glaciology:** Monitoring of ice in the arctic regions, icebergs, and snow coverage.

- **Nautical shipping and economy:** Recording of the sea states, monitoring of shipping lanes, chartering of seaweed appearances.

- **Agriculture and Forestry:** Identification of cultivation types, control of fallowing schemes, monitoring of planting states, observation of grasshopper swarms.

- **Emergency management:** Recognition of surface changes, e.g. cost erosion, detection of earth- and seaquakes, search and rescue of missing people.

- **Environmental monitoring:** In addition to the applications mentioned above: Recognition and imaging of pollution, e.g. oil films on water surfaces, chartering of damage to forests.
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Applications: Analysis of floods

SAR-image of the Elbe river near Dömitz at times of a disastrous flooding.
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Stripmap mode
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Focused SAR as synthetic antenna

- Distance of a point scatterer at \( \rho \) to an aperture coordinate \( \xi \)
  \[ r_0(\xi; \rho) = \sqrt{\rho^2 + \xi^2}. \]

- Phase along the synthetic aperture
  \[ \varphi(\xi; \rho) = -2k_r r_0(\xi; \rho) \]

- Azimuth resolution after phase compensation and integration along synthetic aperture \( L_x \)
  \[ \delta_u = \lambda/(2L_x) \]
  \[ \delta x = \rho \delta_u = \rho \lambda/(2L_x) \]

Maximum length of a synthetic array
\[ L_a = \rho \delta u_a \]
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Focused SAR as synthetic antenna

- Length of the antenna footprint (\(\delta u_a\) denotes the beamwidth of the real antenna):
  \[ L_a = \rho \delta u_a \]

- For fixed antenna look direction the distance the platform is flying from a scatterer entering into the mainbeam until its disappearance is \(L_a\), too. So for this mode, the length of the synthetic array \(L_x\) is bounded by
  \[ L_x \leq L_a \]

- Consequently, the achievable azimuth resolution for stripmap SAR is
  \[ \delta x_{\text{min}} = \rho \frac{\lambda}{2L_a} = \rho \frac{\lambda}{2\rho \delta u_a} = \frac{\lambda}{2\delta u_a} \]

- For a real antenna we have
  \[ \delta u_a = \frac{\lambda}{l_x} \]

- It follows
  \[ \delta x_{\text{min}} = \frac{\lambda}{2\delta u_a} = \frac{\lambda}{2\lambda/l_x} = \frac{l_x}{2} \]
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Focused SAR as synthetic antenna

**Essence**  
The minimum obtainable azimuth resolution of a SAR in stripmap mode with an antenna of length $l_x$ is independent on the range and the wavelength equal to the half length of the antenna.

There are some differences compared to a real array:

- The phase terms always are related to two-ways traveling. Compared to a real array always the double phase has to be inserted. As a consequence, for the same aperture the half beamwidth is obtained. On the other hand, the minimum spatial sampling interval is reduced by the factor 2.

- Omnidirectional antennas would have to be spaced at the raster $\lambda/4$. The phase centers of directed antennas with the real aperture $l_x$ are not allowed to move more than $l_x/2$ from pulse to pulse.

- The pattern of the synthetic antenna shows only to a one-way characteristics, since the way forward and backward had been regarded in the phase terms. Unfortunately, this leads to a higher side lobe level compared to the two-way pattern of a real array!

- Because of the long synthetic aperture a far field focusing is no longer possible.

- The quadratic term of the phase along the synthetic aperture is related to a distinct range. Like for an optical lens a high quality focusing is obtained only in a certain range interval, the depth of focus.
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Depth of focus

The depth of focus can be roughly estimated by the following consideration: The range variation for a scatterer at the slant range $\rho$ is approximately $r_0(\xi; \rho) - \rho \approx \frac{\xi^2}{2\rho}$, that of a scatterer at slant range $\rho \pm d/2$, where $d$ will be the depth of focus, is approximated in an analogue way. To keep the difference between the variations below $\gamma \lambda$, where $\gamma$ is a fraction, e.g. $\gamma = 1/8$, the following inequation has to be fulfilled:

$$\left| \frac{\xi^2}{2\rho} - \frac{\xi^2}{2(\rho \pm d/2)} \right| \leq \gamma \lambda \text{ for all } \xi \in \left[ -\frac{L_x}{2}, \frac{L_x}{2} \right].$$

(1.3)

This leads to the condition

$$\gamma \lambda \geq \left( \frac{L_x}{2} \right)^2 \left| \frac{1}{2\rho} - \frac{1}{2(\rho - d/2)} \right| \approx \left( \frac{L_x}{2} \right)^2 \frac{d}{4\rho^2}$$

(1.4)

and finally to the depth of focus:

$$d_{\text{max}} = \frac{16 \gamma \lambda \rho^2}{L_x^2}.$$  

(1.5)
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Cylinder coordinates

Scene coordinates

Sensor coordinates

Earth plane

Radar

\( \xi = VT \)

\( h \)

\( \rho \)

\( \vartheta \)

\( V \)
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Cylinder coordinates - azimuth/slant range coordinates

- Since the range from the antenna to a scatterer depends only on the azimuth coordinate and the radius $r$, it is convenient to introduce a cylindric coordinate system $(x, \rho, \vartheta)$, where the third parameter, the elevation angle, does not influence the range function.

- It merely modulates the signal amplitude by the antenna characteristics.

- So we can reduce the geometry to two dimensions $(x; \rho)$.
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Two-dimensional coordinates

Point scatterer $(x, \rho)$

$r(\xi; x, \rho)$

$\mathbf{u}(\xi; x, \rho)$

$\alpha(\xi; x, \rho)$

$
\xi(T) = VT$

Motion of SAR sensor
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The azimuth signal

- Range and direction histories

\[ r(\xi; x, \rho) = \sqrt{(x - \xi)^2 + \rho^2}, \quad u(\xi; x, \rho) = \frac{x - \xi}{r(\xi; x, \rho)} \]

\[ u(\xi; x, \rho) = \cos \alpha(\xi; x, \rho) \] is the \(x\)-component of the LOS vector \(\vec{u}(\xi; x, \rho)\)

- We regard a scatterer at \(x = 0\). Representative histories:

\[ r_0(\xi; \rho) := r(\xi; 0, \rho) \quad \text{and} \quad u_0(\xi; \rho) := u(\xi; 0, \rho) \]

\[ r(\xi; x, \rho) = r_0(\xi - x; \rho) \quad u(\xi; x, \rho) = u_0(\xi - x; \rho) \]

\[ r_0(\xi; \rho) = \sqrt{\xi^2 + \rho^2}, \quad u_0(\xi; \rho) = -\frac{\xi}{r_0(\xi; \rho)} \]
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The azimuth signal

Derivatives with respect to the platform position:

\[ r'_0(\xi; \rho) = -u_0(\xi; \rho), \quad u'_0(\xi; \rho) = -\frac{\rho^2}{r_0^3(\xi; \rho)} \]
\[ r''_0(\xi; \rho) = -u'_0(\xi; \rho) \]

Values at \( \xi = 0 \):

\[ r'_0(0; \rho) = 0, \quad u'_0(0; \rho) = -\frac{1}{\rho}, \quad r''_0(0; \rho) = \frac{1}{\rho} \]

Taylor approximations:

\[ r_0(\xi; \rho) \approx \rho + \frac{1}{2} r''_0(0; \rho) \xi^2 = \rho + \frac{1}{2\rho} \xi^2 \]
\[ r'_0(\xi; \rho) \approx \frac{\xi}{\rho} \]

Valid for small angular deviations from the normal direction, where the range hyperbola can sufficiently be approximated by the range parabola.
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The azimuth signal

**Essence 1.2** The azimuth signal of any point scatterer at \((\xi, \rho)\) is a shifted version of the model signal of a point scatterer at \((0, \rho)\). The range hyperbola can be approximated by the range parabola for sufficiently small angular deviations of the normal direction. In this region, the directional cosine can be regarded as a linear function of the position \(\xi\).
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The azimuth chirp

- Due to the range and direction histories we get the following model signal in the slow time domain:

\[ s_0(T) = \exp \left\{ -j2k_r r_0(VT; \rho) \right\} D(u_0(VT; \rho)) \]

- Quadratic approximation of range history:

\[ s_0(T) \approx e^{-j2k_r \rho} \exp \left\{ -j k_r \frac{(VT)^2}{\rho} \right\} D(u_0(VT; \rho)) \]
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The azimuth chirp - Phase and Doppler history

- Phase and Doppler history:

\[
\varphi(\xi; \rho) = -2k_r r_0(\xi; \rho)
\]
\[
F_0(\xi; \rho) = -\frac{1}{2\pi} 2k_r V r_0'(\xi; \rho)
\]
\[
= \frac{2V}{\lambda} r_0'(\xi; \rho)
\]
\[
= F_{max} u_0(\xi; \rho)
\]

\[F_{max} = 2V/\lambda\]

- Quadratic approximation:

\[
\varphi(\xi; \rho) \approx -2k_r (\rho + \frac{1}{2\rho} \xi^2)
\]
\[
F_0(\xi; \rho) \approx -F_{max} \frac{\xi}{\rho}.
\]
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The azimuth chirp

- Time histories of the range (phase), the angle, the Doppler frequency and the azimuth chirp
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Azimuth chirp and synthetic aperture
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Azimuth point spread function by summing up over a synth. aperture without phase compensation ("unfocused SAR")

Increasing length of synthetic aperture
## SYNTHETIC APERTURE RADAR

### Summary of the interdependencies between the variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Position</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>$\equiv$</td>
<td>$r_0(\xi) = \sqrt{\rho^2 + \xi^2}$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>$\xi(r_0) = \pm \sqrt{r_0^2 - \rho^2}$</td>
<td>$\equiv$</td>
</tr>
<tr>
<td>$u_0$</td>
<td>$\xi(u_0) = -\rho \frac{u_0}{\sqrt{1-u_0^2}}$</td>
<td>$r_0(u_0) = \rho \frac{1}{\sqrt{1-u_0^2}}$</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$\xi(F_0) = \rho \frac{F_0}{\sqrt{F_{max}^2 - F_0^2}}$</td>
<td>$r_0(F_0) = \rho \frac{F_{max}}{\sqrt{F_{max}^2 - F_0^2}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Direction</th>
<th>Doppler</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>$u_0(\xi) = -\frac{\xi}{\sqrt{\rho^2 + \xi^2}}$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>$u_0(r_0) = \pm \frac{\sqrt{r_0^2 - \rho^2}}{r_0}$</td>
</tr>
<tr>
<td>$u_0$</td>
<td>$\equiv$</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$u_0(F_0) = \frac{F_0}{F_{max}}$</td>
</tr>
</tbody>
</table>
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The azimuth chirp - important parameters

- Maximum length of the synthetic aperture

\[ L_a = \rho \left( \frac{u_2}{\sqrt{1 - u_2^2}} - \frac{u_1}{\sqrt{1 - u_1^2}} \right) \]

- Duration of illumination

\[ T_s = L_a / V. \]

- Doppler modulation rate

\[ \frac{\partial}{\partial T} F_0(VT; \rho) = F_{max} V u'_0(VT; \rho) \]

\[ = -F_{max} V \rho^2 \frac{r_0^3(VT; \rho)}{r_0^3(VT; \rho)} \]

- For small beamwidth

\[ T_s \approx \delta u_a \frac{\rho}{V} = \frac{\rho \lambda}{l_x V} \]

- At time \( T=0 \):

\[ \beta = \left. \frac{\partial}{\partial T} F_0(VT; \rho) \right|_{T=0} = -F_{max} \frac{V}{\rho} = -\frac{2V^2}{\lambda \rho} \]
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The azimuth chirp - important parameters

- Doppler bandwidth

\[
B = F_{max}(u_2 - u_1) = F_{max} \delta u_a = \frac{2V}{\lambda} \delta u_a
\]

\[
B \approx \frac{2V \lambda}{\lambda l_x} = \frac{V}{l_x/2}
\]

for an antenna with length \( l_x \)

- Time-bandwidth product

\[
BT_s = \frac{L_a 2V}{V} \delta u_a = \frac{2L_a \delta u_a}{\lambda} = \frac{L_a}{l_x/2}
\]

\[
BT_s \approx \frac{2L^2}{\lambda \rho} = \frac{2\lambda \rho}{l_x^2}
\]

- The time-bandwidth product (compression rate) is equal to the ratio between the synthetic and the half real aperture
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The azimuth chirp - important parameters

- Numerical example

<table>
<thead>
<tr>
<th>given</th>
<th>deducted</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>( 150 \text{ m/s} )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( 3 \text{ cm} )</td>
</tr>
<tr>
<td>( l_x )</td>
<td>( 3 \text{ m} )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( 150 \text{ km} )</td>
</tr>
<tr>
<td></td>
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</tbody>
</table>

**Essence**

The azimuth signal has the form of a chirp over slow-time with approximately linear frequency modulation around the direction perpendicular to the flight direction, multiplied with the two-way characteristics of the antenna in the instantaneous direction to the scatterer passing by. It is called 'azimuth chirp'.
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The flying radar as a (slow-)time-invariant linear filter

- Measured signal for azimuth reflectivity normalized to slow time \( \tilde{a}(T) = a(\nu T) \)

\[
z(T) = \int s_0(T - T')\tilde{a}(T')dT' + n(T) = x(T) + n(T)
\]

- The flying radar acts on the azimuth reflectivity signal like a time-invariant linear filter, whose pulse response is \( s_0(T) \).

- Transfer function: \( S_0(F) \)

\[
X(F) = S_0(F)\tilde{A}(F)
\]

*Essence* The moving radar responds to the azimuth reflectivity like a time-invariant linear filter with pulse response \( s_0(T) \) and transfer function \( S_0(T) \).
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Azimuth compression

- To compress the azimuth signal, we use a filter with pulse response $h(T)$

- Analogue to the pulse compression in fast time, the point spread function in azimuth is the response of the compression filter to the azimuth model signal:

$$p(T) = (h \ast s_0)(T), \quad P(F) = H(F)S_0(F)$$

- The SNR-optimum compression filter is the matched filter, given by

$$h_{mf}(T) = s_0^*(-T) \quad H_{mf}(F') = S_0^*(F')$$

- The point spread function in azimuth is the Fourier back-transform of

$$P_{mf}(F) = |S_0(F)|^2$$
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Azimuth compression - inverse filter

- Inverse filter over a Doppler-frequency set $F$ with $S_0(F)$ not equal to 0.

\[ H_{inv}(F) = \begin{cases} 
\frac{1}{S_0(F)} & \text{if } F \in F \\
0 & \text{else.} 
\end{cases} \]

\[ P_{inv}(F') = I_F(F') \frac{1}{S_0(F)} S_0(F)(F) = I_F(F) \]

**Essence**  
If the azimuth compression filter is chosen as the matched filter, the azimuth point spread function is the autocorrelation function of the azimuth chirp, it is the Fourier back transform of the square magnitude of the azimuth chirp spectrum. If the compression filter is chosen as the inverse filter, the point spread function is the Fourier back transform of the Indicator function of the Doppler frequency band.
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Evaluation of the azimuth chirp - spectrum

Rough estimate of the spectral power:

- Direction related to Doppler F, power from this direction
  \[ u_0(F) = \frac{F}{F_{\text{max}}} \quad P(u_0) = |D(u_0)|^2 \]

- So we get approximately
  \[ |S_0(F')|^2 \sim |D(u_0(F'))|^2 = \left| D \left( u_0 \left( \frac{F}{F_{\text{max}}} \right) \right) \right|^2 \]

- Again, the Doppler power spectrum is a scaled version of the two-ways antenna pattern, as similar stated for the clutter spectrum before.

- The *clutter spectrum is* generated by many ground based scatterers at different azimuth positions, observed over a short time.

- Here, we assumed a single scatterer observed for the whole time of illumination.
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Chirp selection theorem

The integral of a chirp (see 'Anatomy of a chirp'):

$$\int e^{j\pi \alpha t^2} dt = \frac{1 + j}{\sqrt{2\alpha}}$$

For a smooth signal $s(t)$ we get:

$$\int s(t)e^{j\pi \alpha t^2} dt \approx s(0) \int e^{j\pi \alpha t^2} dt$$

$$= s(0) \frac{1 + j}{\sqrt{2\alpha}}$$

Transferred to $t=t_0$:

$$\frac{\sqrt{2\alpha}}{1 + j} \int s(t)e^{j\pi \alpha(t-t_0)^2} dt \approx s(t_0)$$

More general:

$$\lim_{\alpha \to \infty} \frac{\sqrt{2\alpha}}{1 + j} e^{j\pi \alpha t^2} = \delta(t)$$

(Chirp selection theorem)

Main contribution to the integral
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Principle of stationary phase

We regard an integral of the form

\[ I = \int_{a}^{b} s(t)e^{i\phi(t)} dt \]

Let \( s(t) \) be slowly varying and \( \Phi(t) \) two times differentiable and running through many \( 2\pi \) intervals.

Let be a unique \( t_0 \) with

\[ \frac{d}{dt} \phi(t) \big|_{t=t_0} = 0 \]

\( t_0 \) is called 'point of stationary phase'.

We can expand the phase around \( t=t_0 \):

\[ \phi(t) \approx \phi(t_0) + \pi \alpha(t - t_0)^2 \]

\[ \alpha = \frac{1}{2\pi} \frac{d^2}{dt^2} \phi(t) \big|_{t=t_0}. \]

Then:

\[ \int_{a}^{b} s(t)e^{i\phi(t)} dt \approx s(t_0)e^{i\phi(t_0)} \int e^{i\pi \alpha(t-t_0)^2} dt \]

\[ = s(t_0)e^{i\phi(t_0)} \frac{1 + j}{\sqrt{2\alpha}}. \]

For more than one point of stationary phase, the contributions sum up.
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Application to the Fourier transform

We regard a signal of the form

\[ z(t) = e^{j\phi(t)} s(t) \]

with slowly varying \( s(t) \) and a phase \( \phi(t) \)
with the above mentioned properties, and compute the Fourier transform:

\[
Z(f) = \int e^{-j2\pi ft} e^{j\phi(t)} s(t) dt \\
\approx e^{j(\phi(t_0)-2\pi ft_0)} s(t_0) \frac{1 + j}{\sqrt{2\alpha}} \\
= e^{-j2\pi ft_0} z(t_0) \frac{1 + j}{\sqrt{2\alpha}},
\]

with \( t_0 \) determined by

\[ \frac{d}{dt} \phi(t) \big|_{t=t_0} = 2\pi f \]

and

\[ \alpha = \frac{1}{2\pi} \frac{d^2}{dt^2} \phi(t) \big|_{t=t_0} \]

Remark, that we spare an integral with this approximation!
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Fourier transform of the azimuth chirp

\[ S_0(F) = \int e^{-j2\pi FT} s_0(T) dT \]

\[ = \int e^{-j2\pi FT} \exp \left\{ -j2k_r r_0(VT; \rho) \right\} D(u_0(VT; \rho)) dT \]

- The phase:

\[ \phi(T) = -2\pi FT + \varphi(T) \text{ with } \varphi(T) = -2k_r r_0(VT; \rho) \]

**Problem** Determine the point of stationary phase \( T_0 \), the phase \( \phi(T_0) \), and its second derivative at this point!

\[ 0 = -2\pi F + \frac{d}{dT} \varphi(T)|_{T=T_0} \]

\[ F = F_0(VT_0; \rho). \]
SYNTHETIC APERTURE RADAR
Fourier transform of the azimuth chirp

So the principle of stationary phase demands that the point of time $T = T_0$ is found where the instantaneous Doppler frequency $F_0(VT_0; \rho)$ is equal to the frequency $F$ of the Fourier cell.

$$2\pi F = \frac{d}{dT} \varphi(T) \bigg|_{T=T_0} = 2k_r V u_0$$

The value of the stationary phase:

$$\phi(T_0) = -2\pi F T_0 + \varphi(T_0) = -2k_r V T_0 u_0 + \varphi(T_0)$$

$$= -2k_r \xi_0 u_0 - 2k_r r_0 = 2k_r r_0 u_0^2 - 2k_r r_0$$

$$= -2k_r r_0 (1 - u_0^2) = -2k_r \rho \sqrt{1 - u_0^2}$$

$$= -2k_r \rho \sqrt{1 - \left( \frac{F}{F_{\text{max}}} \right)^2}$$
SYNTHETIC APERTURE RADAR
Fourier transform of the azimuth chirp

Second derivative of the phase:

\[
\frac{d^2}{dT^2} \phi(T)|_{T=T_0} = 2k_r V^2 u_0' = -2k_r V^2 \frac{\rho^2}{r_0^3}
\]

\[
= -2k_r V^2 (1 - u_0^2)^{3/2} \frac{1}{\rho}
\]

\[
= -\frac{2k_r V^2}{\rho} \left(1 - \left(\frac{F}{F_{\text{max}}}\right)^2\right)^{3/2}
\]
SYNTHETIC APERTURE RADAR

Fourier transform of the azimuth chirp

Result:

\[ S_0(F) \approx C(F) \exp \left\{ -j2k_r\rho \sqrt{1 - \left( \frac{F}{F_{\text{max}}} \right)^2} \right\} D \left( \frac{F}{F_{\text{max}}} \right) \]

\[
C(F') = \frac{1 + j}{\sqrt{2\alpha(F)}}
\]

\[
\alpha(F') = \frac{1}{2\pi} \frac{2k_rV^2}{\rho} \left( 1 - \left( \frac{F}{F_{\text{max}}} \right)^2 \right)^{3/2}
\]

**Essence** The power spectrum of the azimuth chirp is a scaled version of the magnitude-square of the two-way antenna characteristics. The complex azimuth point target reference spectrum \( S_0(F) \) can be approximated by the principle of the stationary phase by above equations. This approximation can be used to perform the azimuth compression in the Doppler domain.
SYNTHETIC APERTURE RADAR

Compression of the azimuth chirp - compression filters

- The SNR-optimum azimuth compression filter is the matched filter

\[ h_{mf}(T) = s_0^*(-T) \]

- The point spread function is the inverse Fourier transform of

\[ P_{mf}(F) = |S_0(F)|^2 \]

- Window in the slow-time domain, or in the Doppler domain:

\[ h_w(T) = s_0^*(T)w_T(T), \quad H_w(F) = S^*(F)w_F(F) \]

- Inverse filter over a set \( F \) of frequencies with non-vanishing signal spectrum:

\[ H_{inv}(F) = \begin{cases} \frac{1}{S_0(F)} & \text{if } F \in F, \\ 0 & \text{else.} \end{cases} \]
SYNTHETIC APERTURE RADAR

Compression of the azimuth chirp - compression filters

- Fourier transform of the point spread function for inverse filtering:

\[ P_{\text{inv}}(F) = I_F(F) \frac{1}{S_0(F)} S_0(F')(F') = I_F(F) \]

- Fourier transform of the azimuth reflectivity scaled to slow-time: \( \tilde{A}(F') \)

- Fourier transform of the compressed scene \( i \)

\[ Y_{\text{inv}}(F) = P_{\text{inv}}(F') \tilde{A}(F') = I_F(F') \tilde{A}(F') \]

**Essence**

If the azimuth compression filter is chosen as the matched filter, the azimuth point spread function is the autocorrelation function of the azimuth chirp, it is the Fourier back transform of the square magnitude of the azimuth chirp spectrum. If the compression filter is chosen as the inverse filter, the point spread function is the Fourier back transform of the Indicator function of the Doppler frequency band.
SYNTHETIC APERTURE RADAR

Compression of the azimuth chirp - Spotlight and sliding mode

- For the **stripmap mode** the antenna must not be longer than twice the desired azimuth resolution. That means that a very short antenna has to be used to obtain a fine resolution. But this is a problem in several aspects:
  - The antenna gain will be low leading to a poor SNR.
  - The Nyquist condition for azimuth sampling (at least one pulse per movement of the half antenna length) requires a high PRF leading to a high data rate and perhaps range ambiguities.

- In the **spotlight mode** the antenna beam is steered to a fixed point at the earth. In this way the synthetic aperture can be elongated nearly arbitrarily to achieve fine resolution. The instantaneous Doppler bandwidth is still given by $B = 2V/l_x$ and the PRF has to be adopted to this bandwidth, while the center Doppler frequency changes according to the varying look direction. A disadvantage of the spotlight mode is the restriction of the azimuth length of the scene to the width of the antenna footprint.
SYNTHETIC APERTURE RADAR

Compression of the azimuth chirp - Spotlight and sliding mode

- A continuous intermediate state between stripmap and spotlight mode can be obtained by the **sliding mode**.

- Here the center point of the antenna footprint moves along the earth surface with a velocity which is normally between zero and the platform velocity, but sometimes it is also desirable to let the footprint slide over the earth with a higher velocity or even opposite to the flight direction. In all these cases, a feasible balance between azimuth extension of the scene and azimuth resolution can be achieved.
SYNTHETIC APERTURE RADAR

The signal in the $\xi$ - r domain

- We go back to the original raw data before pre-processing to the normal form. If the transmit signal $s(r)$ - written as function of the spatial variable $r$ - is used, the following raw data signal in the $(r,\xi)$-plane from a point scatterer at the position $(x,\rho)$ is got:

$$s_{\xi,r}(\xi, r; x, \rho) = s\left(r - r_0(\xi - x; \rho)\right)$$

$$\times \exp \left\{ -j2k_0r_0(\xi - x; \rho) \right\} D(u_0(\xi - x; \rho))$$

- Now we consider a chirp as transmit signal, written in the spatial variable as

$$s(r) = \text{rect} \left( \frac{r}{r_s} \right) \exp \left\{ j\alpha\pi \left( \frac{r}{r_s} \right)^2 \right\}$$
SYNTHETIC APERTURE RADAR

The signal in the $\xi$ - r domain

- Signal if the wave form is a chirp:

$$s_{\xi,r}(\xi, r; x, \rho) = \text{rect} \left( \frac{r - r_0(\xi - x; \rho)}{r_s} \right) \exp \left\{ j\alpha \pi \left( \frac{r - r_0(\xi - x; \rho)}{r_s} \right)^2 \right\}$$

$$\times \exp \left\{ -j2k_0r_0(\xi - x; \rho) \right\} D\left( u_0(\xi - x; \rho) \right)$$

- The phase of this expression (the approximation is valid in the vicinity of $(\rho, x)$)

$$\varphi(\xi, r) = \alpha \pi \left( \frac{r - r_0(\xi - x; \rho)}{r_s} \right)^2 - 2k_0r_0(\xi - x; \rho)$$

$$\approx \alpha \pi \left( \frac{r - \rho}{r_s} \right)^2 - 2k_0 \left( \rho + \frac{1}{2\rho}(\xi - x)^2 \right)$$

$$= -2k_0\rho + \frac{\alpha \pi}{r_s^2}(r - \rho)^2 - \frac{2k_0}{2\rho}(\xi - x)^2$$
SYNTHETIC APERTURE RADAR
The signal in the $\xi$ - r domain

- Since the exponents, which are both approximately quadratic in their respective variables add, the lines of equal phases are approximately ellipses, if the signs of the quadratic terms coincide, otherwise they are hyperbolas. Since the azimuth chirp is always a down-chirp, ellipses appear if the transmit signal is also a down chirp, and hyperbolas, otherwise.

- Around a fixed point targets we observe in the first case a sequence of concentric ellipses, which have the same sequence of radius as the fringes of a Fresnel zone plate which are the cuts of concentric sphere shells with equidistantly growing radius.

- These interference fringes are known from holography. Since such Fresnel interference patterns can be focused using coherent light, can SAR raw data, exposed on a photographic film, be focused to an image using optical methods, as it was done in the early period of SAR processing.
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Signal in the azimuth-range plane (down-chirp)
SYNTHETIC APERTURE RADAR
Real raw data in the azimuth-range domain
SYNTHETIC APERTURE RADAR

The signal in the $\xi - k_r$ - domain

- After the application of a Fourier transform from the $r$- to the $k_r$-dimension followed by an inverse filtering, the signal assumes the form

$$s^{\xi, k_r} (\xi, k_r; x, \rho) = \exp \left\{ -j2k_r r_0 (\xi; \rho) \right\} D (u_0 (\xi; \rho))$$

- The phase of the azimuth chirp is scaled with $k_r$!
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Signal in the $\xi - k_r$ plane
SYNTHETIC APERTURE RADAR

The signal in the $k_x$ - $k_r$ domain

- A spatial Fourier transform along $\xi$ yields

$$s^{k_x,k_r}(k_x, k_r; x, \rho) = e^{-j k_x x} s^{k_r,k_x}(k_r, k_x; 0, \rho)$$

$$= e^{-j k_x x} \int e^{-jk_x \xi} \exp \{ -j 2k_r r_0(\xi; \rho) \} D(u_0(\xi; \rho)) \, d\xi$$

- Again, the principle of stationary phase can be applied. The complete phase is given by

$$\phi(\xi) = -k_x \xi - 2k_r r_0(\xi; \rho)$$
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The signal in the $k_x - k_r$ domain

- For each $k_x$, a position $\xi_0$ has to be determined where the derivation of the phase history is zero, resulting in

\[
0 = -k_x - 2k_r r'_0(\xi_0; \rho) \\
\implies k_x = 2k_r u_0(\xi_0; \rho)
\]

- The right side of this equation is the phase gradient along the synthetic aperture. Obviously, that position $\xi_0$ is searched for which this phase gradient is equal to $k_x$. 
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The signal in the $k_x - k_r$ domain

- Since the absolute value of the phase gradient is always lower or equal to $2k_r$, we can characterize $k_x$ as the abscissa component of a vector with length $2k_r$ in the k-domain $(k_x; k_r)$.

- In this plane, $k_x = 2k_r \cos \beta$, where $\beta$ is the angle from the $k_x$-axis to the vector.

\[
\cos \beta = u_0(\xi_0; \rho) = \cos \alpha
\]
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The signal in the $k_x$ - $k_r$ domain - geometrical interpretation

- The phase can be reduced to a length of a way.

\[
\phi(\xi) = -k_x \xi - 2k_r r_0(\xi; \rho) \\
\phi(\xi) = -2k_r g(\xi) \\
g(\xi) = \xi \cos \beta + r_0(\xi; \rho)
\]
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The signal in the $k_x - k_r$ domain - geometrical interpretation
SYNTHETIC APERTURE RADAR

The signal in the $k_x$ - $k_r$ domain - geometrical interpretation

- Computation of the minimum way $g(\xi_0)$.

$$\xi_0 = -\rho \cot \beta,$$
$$r_0(\xi_0; \rho) = \frac{\rho}{\sin \beta}.$$

Also the value of $g(\xi_0)$ is easily obtained:

$$g(\xi_0) = \rho \sin \beta$$
$$= \rho \sqrt{1 - \cos^2 \beta}$$
$$= \rho \sqrt{1 - \frac{k_r^2}{4k_x^2}}$$
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The resulting phase - interpreted as the Thales half circle

\[ \phi(\xi_0) = -2k_r g(\xi_0) = -\rho \sqrt{4k_r^2 - k_x^2}. \]
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The signal in the $k_x - k_r$ domain
SYNTHETIC APERTURE RADAR

The signal in the $k_x$ - $k_r$ domain

**Essence**  
*Using the principle of the stationary phase, the phase of the Fourier transform of the azimuth chirp in dependence on the spatial frequency along the azimuth can be deduced to the cathetus length in the half circle of Thales as illustrated.*

- Computation of the second derivative of the phase.

\[
\phi''(\xi_0) = -2k_rr''_0(\xi_0; \rho) \\
= -2k_r \rho^2 r_0^3(\xi_0; \rho) \\
= -2k_r \rho^2 (\rho / \sin \alpha)^3 \\
= -2k_r \sin^3 \alpha / \rho \\
= -2k_r \rho \left( 1 - \frac{k_x^2}{4k_r^2} \right)^{3/2} \\
= -\frac{1}{\rho 4k_r^2} \left( 4k_r^2 - k_x^2 \right)^{3/2}.
\]
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The signal in the $k_x$ - $k_r$ domain

Resulting formula for the signal in this domain

$$s^{k_x,k_r}(k_x, k_r; x, \rho) \approx C(k_x) \exp\left\{-j \left(k_x x + \rho \sqrt{4k_r^2 - k_x^2}\right)\right\} D \left(\frac{k_x}{2k_r}\right)$$

$$C'(k_x) = (1 + j) \sqrt{\frac{\pi \rho 4k_r^2}{(4k_r^2 - k_x^2)^{3/2}}}.$$
SYNTHETIC APERTURE RADAR

K-set and point spread function

- Platform position $\vec{R}(\xi)$
- Reference point at the surface $\vec{p}_0$
- Point in the neighborhood of the reference point $\vec{p} = \vec{p}_0 + \vec{p}'$

- Local approximation of the range history

$$r(\xi; \vec{p}) = \| \vec{R}(\xi) - \vec{p} \|$$
$$\approx r(\xi; \vec{p}_0) + \langle \vec{u}_0(\xi), \vec{p}' \rangle$$

with $\vec{u}_0(\xi) = \frac{\vec{R}(\xi) - \vec{p}_0}{|\vec{R}(\xi) - \vec{p}_0|}$ pointing to the reference point
SYNTHETIC APERTURE RADAR

K-set and point spread function

- Signal in the neighborhood of the reference point
  (The two-way characteristics is written as function of the double k-vector)

\[ s(\xi, k_r; \vec{p}') = e^{-j2k_r r(\xi; \vec{p}_0)} \exp\{-j2k_r \langle \vec{u}_0(\xi), \vec{p}' \rangle\} D(2k_r \vec{u}_0(\xi)) \]

- Measurements

\[ z(\xi, k_r) = \int a(\vec{p}') s(\xi, k_r; \vec{p}') d\vec{p}' \]

where \( a \) describes the re\oe\ectivity in the vicinity of the reference point.
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K-set and point spread function

- Phase compensation with respect to the reference point

\[ z_{mc}(\xi, k_r) := e^{j2k_r r(\xi; \vec{p}_0)} z(\xi, k_r) \]

\[ = \int a(\vec{p}') s_{mc}(\xi, k_r; \vec{p}') d\vec{p}' \]

with \( s_{mc}(\xi, k_r; \vec{p}') = e^{j2k_r r(\xi; \vec{p}_0)} s(\xi, k_r; \vec{r}') \)

\[ = \exp\{-j2k_r \langle \vec{u}_0(\xi), \vec{p}' \rangle\} D(2k_r \vec{u}_0(\xi)) \]

- SAR processing: correlation with the model signal given above, normalization

\[ \hat{a}(\vec{p}') = \frac{\int \int z_{mc}(\xi, k_r) \exp\{j2k_r \langle \vec{u}_0(\xi), \vec{p}' \rangle\} D(2k_r \vec{u}_0(\xi)) d\xi dk_r}{\int \int |D(2k_r \vec{u}_0(\xi))|^2 d\xi dk_r} \]
SYNTHETIC APERTURE RADAR

K-set and point spread function

- By variation of \( k_r \) in the interval \( k_r \in [k_{r1}, k_{r2}] \) the variation of the platform position within \( \xi \in [\xi_1, \xi_2] \) a set of wave number vectors is generated:

\[
K = \{ 2k_r \vec{u}_0(\xi) : k_r \in [k_{r1}, k_{r2}], \xi \in [\xi_1, \xi_2] \}
\]

- The k-set is a circular ring segment (see Figure).

\[
\vec{K} := 2k_r \vec{u}_0
\]

- The generated SAR image can be written as reconstruction over this k-set:

\[
\hat{a}(\vec{p}') = \frac{\int_K e^{i(\vec{K} \cdot \vec{p}')}}{\int_K |D(\vec{K})|^2} D(\vec{K}) d\vec{K}
\]
SYNTHETIC APERTURE RADAR

K-set and point spread function

- For rectangular main beam the circular ring segment can be restricted to the main beam, leading to the k-set $\mathcal{K}_0$

- We get the simplified reconstruction

$$\hat{a}(\vec{p}') = \frac{\int_{\mathcal{K}_0} e^{ij\langle \vec{K}, \vec{p}' \rangle} z_{mc}(\vec{K}) d\vec{K}}{\int_{\mathcal{K}_0} d\vec{K}}$$

- Obviously the reconstruction is obtained by the Fourier transformation of the motion compensated data over the k-set.
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Fourier transf.

k-set

k_min

k_max

point spread function
SYNTHETIC APERTURE RADAR

K-set and point spread function

If for $z_{mc}$ the motion compensated signal of a point scatterer at the reference point is inserted, we get the point spread function

$$p(\vec{p}') = \frac{\int_{K_0} e^{j\langle \vec{K},\vec{p}' \rangle} d\vec{K}}{\int_{K_0} d\vec{K}}$$

Essence: The point spread function is the Fourier transform of the indicator function of the k-set.
SYNTHETIC APERTURE RADAR

Generic SAR processor

- The measured signal $z(\xi, r)$ consists of the superposition of the echoes of many point scatterers with the reflectivity at $a(x, \rho)$:

$$z(\xi, r) = \int \int a(x, \rho) s_0(\xi, r; x, \rho) dx d\rho,$$

- After transformation into the $(\xi, k_r)$-domain

$$z^{\xi, k_r}(k_r, \xi) = \int \int a(x, \rho) s^{\xi, k_r}(\xi, k_r; x, \rho) dx d\rho.$$
SYNTHETIC APERTURE RADAR

Generic SAR processor

- Principally, the raw data could be processed to an image optimally, if for each possible position of a point scatterer the two-dimensional normalized matched filter for the model signal would be applied:

Fast time domain:

$$\hat{a}(x, \rho) = \frac{\int \int s^*(\xi, r; x, \rho) z(\xi, r) dr d\xi}{\int \int |s(\xi, r; x, \rho)|^2 dr d\xi}$$

Range frequency domain:

$$\hat{a}(x, \rho) = \frac{\int \int s^{\xi,kr^*}(\xi, kr; x, \rho) z^{\xi,kr}(\xi, kr) dr d\xi}{\int \int |s^{\xi,kr}(\xi, kr; x, \rho)|^2 dr d\xi}$$

- The computational effort would be much too high. The first approach is to apply separated processing, i.e. range compression and azimuth compression are applied one after the other.
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Principal processing
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The range-Doppler processor

- The range-Doppler processor starts with the classical range compression as a rule as 'fast correlation' performed in the range frequency domain.

- Now the effect that the signal runs along a hyperbolic curve through the range cells - the range curvature problem - has to be solved.

- There is a possibility to reduce the numerical effort of the described interpolation along the range hyperbolas:

- First the data are transformed by an azimuth FFT into the $k_x$ domain. The signals of scatterers with the same $\rho$ but varying $x$-coordinates are shifted in slow time; but by the Fourier transform the pathes in the $k_x$-domain are overlayed. For this reason, the range curvature correction can be performed in the $k_x$-domain simultaneously.
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Range curvature correction

Figure 1.23: Range curvature over $k_x$; Center: After correction; right: perfect correction
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The range-Doppler processor
SYNTHEtic aperture Radar

The range-migration processor (Omega-k processor)

- The main steps in the range-migration processor are done in the double-frequency domain.
- Remember:

\[ s^{k_x, k_r}(k_x, k_r; x, \rho) \approx C(k_x) \exp \left\{ -j \left( k_x x + \rho \sqrt{4k_r^2 - k_x^2} \right) \right\} D \left( \frac{k_x}{2k_r} \right) \]

- The measured data are the superposition of these signals with the azimuth reflectivity:

\[ z^{k_x, k_r}(k_x, k_r) = \int \int s^{k_x, k_r}(k_x, k_r; x, \rho) a(x, \rho) dxd\rho \approx C(k_x) \int \int \exp \left\{ -j \left( k_x x + \rho \sqrt{4k_r^2 - k_x^2} \right) \right\} D \left( \frac{k_x}{2k_r} \right) a(x, \rho) dxd\rho. \]
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The range-migration processor (Omega-k processor)

- Variable substitution:
  \[ k_\rho := \sqrt{4k_r^2 - k_x^2} \]

- We get in the new \((k_x, k_\rho)\)-domain:
  \[
z^{k_x, k_\rho}(k_x, k_\rho) \approx C(k_x) D \left( \frac{k_x}{\sqrt{k_x^2 + k_\rho^2}} \right) \int \int \exp \left\{ -j \left( k_x x + k_\rho \rho \right) \right\} a(x, \rho) dx d\rho.
  
  = C(k_x) D \left( \frac{k_x}{\sqrt{k_x^2 + k_\rho^2}} \right) A(k_x, k_\rho)
  
- with \(A(k_x, k_\rho)\) being the two-dimensional Fourier transformation of the reflectivity.

- This substitution has changed the data in the \((k_x, k_r)\)-domain into the 2D Fourier transformation of the reflectivity distribution, modulated by the antenna characteristics.

- The grid in \((k_x, k_r)\) has to be transferred into a grid in the \((k_x, k_\rho)\)-domain. For this the Stolt-interpolation is applied. At the end only a two-dimensional inverse Fourier transformation has to be performed to get the focused SAR image.
SYNTHEtic aperture radar

The range-migration processor (Omega-k processor)
SYNTHETIC APERTURE RADAR

The back-projection processor

For each column contribution to image by interpolation

Accumulation to final image
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Effects in SAR imaging