1. Introduction And Motivation

For real-time visualization of time-variant processes, e. g.
- heart motion,
- blood and contrast agent flow,
- trajectory of propagating shear waves,
the acquisition time for a single ultrasound image is crucial. This time depends significantly on the number of sequentially emitted sound waves. For fewer wave emissions the inverse scattering problem is increasingly ill-posed and cannot be solved uniquely without imposing additional constraints. Recently, compressive sensing (CS) has been introduced as a concept for the fast acquisition of compressible noisy signals [1]. Assuming sparsity of a signal in an arbitrary basis, the aim of CS is to reconstruct the signal from only few physical measurements [2], [1].

In this contribution we establish and investigate a solution to the inverse scattering problem based on CS. Our approach for the lack of measurement data by assuming sparsity of the material parameters in a suitable basis.

2. Solution To The Wave Equation

Classical scan configuration in two-dimensional pulse-echo ultrasound imaging:

Inhomogeneous object
incident transducer array (N elements)

Free-space Green's function:

\[ G(r) = \frac{1}{4 \pi^2} H^0_n(\kappa) \frac{1}{r} \]

Object: \( \Omega \subset \mathbb{R}^2 \) location: \( r \in \mathbb{R}^2 \)
compressibility: relative deviation:

\[ \kappa(r) = \kappa_0 r \in \Omega, \quad \kappa(r) \neq \kappa_0 \quad \gamma(r) = 1 - \frac{\kappa(r)}{\kappa_0} \]

Monochromatic acoustic pressure \( p \) at wavenumber \( k \) is governed by [4]:

\[ (\Delta + k_n^2)p(r) = G(r) \delta(r) \]

Decomposition:

\[ p = p_0 + p_e \]

Incident pressure: \( (\Delta + k_n^2)p_e(r) = 0 \)

Unique solution for scattered pressure [5] satisfies:

\[ p_s(r) = \int_0^\infty \gamma(t)^2(t)G(t - r)dt \]

3. Inverse Scattering Problem

Underlying assumptions:

1. Object consists of \( N = \sum x \) point scatterers on a lattice:

\[ \gamma(x) = \sum_{n=0}^x \delta(x) \quad \text{distances:} \quad \delta_n \quad \text{origin:} \quad (x_0, z_0) \]

2. Plane wave excitation:

\[ p_x(r, e) = A_e(k_0) e^{-i k_0 z} \]

3. Linear scattering according to first Born approximation [5]:

\[ p_s(r, \kappa) = k_n^2 A_e(k_0) \sum_{n=0}^x e^{-i k_0 z} g(r - r_n) \]

4. Experimental Setup

Phantoms:

- sparse object: four wires immersed in water reservoir
- non-sparse object: CIRS model 040 multi-purpose ultrasound phantom (Computerized Imaging Reference Systems, Norfolk, Virginia, USA), discrete cosine transform (DCT) as sparsifying transform

Data acquisition:

- Ultrasonix SonataTouch Research System (Ultrasonix Medical Corp., Richmond, BC, Canada) with L4-5/38 linear array transducer (\( N = 128 \) elements)
- Full synthetic aperture (SA) scans [7] utilizing 128 single element emissions
- Averaged and filtered SA data is utilized to compute reference images and to synthesize plane wave measurement data with \( e_s = e_0 \).

Algorithms:

- Synthetic aperture reconstruction (SAR) [7]: 128 single element emissions
- Delay-and-sum (DAS) beamforming: single plane wave emission
- Filtered backpropagation (FBP) [8]: single plane wave emission

5. Experimental Results (Wire Phantom)

Details of the images:

- Axial and lateral image profiles:
- Axial and lateral profiles: FBP (c), CS (d)
- Lateral -6 dB-widths of CS are clearly smaller than those of SA, DAS and FBP.

6. Experimental Results (Tissue Phantom)

Details of the images:

- Axial and lateral profiles: FBP (a), CS (d)
- Axial and lateral profiles: SA (b), DAS (c)
- Lateral -6 dB-widths of CS are clearly smaller than those of SA, DAS and FBP.

7. Conclusion And Outlook

We investigated the performance of compressive sensing (CS) in solving the inverse scattering problem arising in fast pulse-echo ultrasound imaging.

- Wire phantom (sparse object): CS yields the best results in terms of sidelobe reduction and lateral -6 dB-widths utilizing only a single plane wave emission.
- Tissue phantom (non-sparse object): CS yields similar results as delay-and-sum (DAS) beamforming and filtered backpropagation (FBP) for a single plane wave emission.

The performance of CS for non-sparse objects can probably be improved by utilizing a more suitable sparsifying transform, e. g. Shannon wavelets. Our model can also be adapted to the excitation with spherical waves.

References


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